GENERALIZATION OF DATA ON CONVECTIVE HEAT TRANSFER WITH VAPORIZATION TO A TURBULENT BOUNDARY LAYER

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Dimensionless complexes describing heat transfer from a plate on evaporation from the surface are derived on the basis of the integral momentum and energy relationships for a turbulent boundary layer.

To the present there exists no rigorous solution of the problem of transfer in a turbulent boundary layer [1]. Integral relationships for the boundary layer in terms averaged over time and over the transverse coordinate have gained widespread acceptance. For the case of gradient-free streamlining of a flat wall by a transverse flow of matter, the problem is formulated in the form

$$\frac{d\delta_H^{**}}{dx}-\frac{c_{pw}\rho_w u_w}{c_{p\infty}\rho_\infty V_\infty}=\frac{\dot q_w''}{\rho_\infty V_\infty (H_\infty - H_w)}, \qquad (1)$$

$$\frac{d\delta^{**}}{dx}-\frac{\rho_w u_w}{\rho_\infty V_\infty}=\frac{\tau}{\rho_\infty V_\infty^2}, \qquad (2)$$

where $\delta_H^{**}=\int_0^\delta \frac{\rho u}{\rho_\infty V_\infty}\left(1-\frac{H-H_w}{H_\infty-H_w}\right)dy$ is the thermal energy (enthalpy) thickness; $\delta^{**}=\int_0^\delta \frac{\rho u}{\rho_\infty V_\infty}\times\left(1-\frac{u}{V_\infty}\right)dy$ is the momentum thickness.

Let us use the generally accepted terms [2]: $c_f = 2\tau_w/\rho V_\infty^2$ and $\mathrm{St} = \dot q_w''/\rho_\infty V_\infty (H_\infty - H_w)$. If we keep to the hypothesis of similarity for the dynamic and enthalpy (temperature) profiles of the boundary layer, we have the equality $\mathrm{St} = c_f/2$. Dividing (1) and (2) by St_0, term by term, we obtain

$$\frac{1}{\mathrm{St}_0}\frac{d\delta_H^{**}}{dx}-b_H=\frac{\mathrm{St}}{\mathrm{St}_0}, \qquad (3)$$

$$\frac{1}{\mathrm{St}_0}\frac{d\delta^{**}}{dx}-b=\frac{\mathrm{St}}{\mathrm{St}_0}, \qquad (4)$$

where

$$b_H=\frac{c_{pw}\rho_w u_w}{c_{p\infty}\rho_\infty V_\infty \mathrm{St}_0}; \qquad (5)$$

$$b=\frac{\rho_w u_w}{\rho_\infty V_\infty \mathrm{St}_0}=\frac{c_{pw}}{c_{p\infty}}b_H; \qquad (6)$$

St_0 is the dimensionless coefficient of heat transfer for an impermeable surface for the same thermal and hydrodynamic conditions.

The parameters b_H and b, characterizing the effect of mass transfer on the transfer of heat, have gained extensive acceptance in the study of problems pertaining to the turbulent boundary layer at a permeable surface (injection—removal of a gas).

Let us express b_H and b in terms of quantities characterizing the process of liquid evaporation at the streamlined wall. Let us perform certain transformations and substitutions, bearing in mind that

$$\rho_w u_w = \dot m'' = \mathrm{Nu}\,\frac{\lambda}{l}\,\frac{T_\infty - T_w}{r + c_1 (T_w - T_1)}, \qquad (7)$$

$$\mathrm{St}_0=\frac{\mathrm{Nu}_0}{\mathrm{RePr}}, \qquad (8)$$

$$\mathrm{Re}=\frac{V_\infty l}{\nu}, \qquad (9)$$

$$\mathrm{Pr}=\frac{\nu}{a}=\frac{\nu\rho_\infty c_{p\infty}}{\lambda}, \qquad (10)$$

in which case

$$b_H=\frac{\mathrm{Nu}}{\mathrm{Nu}_0}\,\frac{c_{pw}}{c_1}\left[\frac{r}{c_1(T_\infty - T_w)}+\frac{T_w - T_1}{T_\infty - T_w}\right]^{-1}. \qquad (11)$$

The term $r/c_1(T_\infty - T_w) = K$ represents the criterion of phase conversion.

It follows from (11) that

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_0}=b_H\frac{c_1}{c_{pw}}\left(K+\frac{T_w - T_1}{T_\infty - T_w}\right) \qquad (12)$$

or

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_0}=b\,\frac{c_1}{c_{p\infty}}\left(K+\frac{T_w - T_1}{T_\infty - T_w}\right). \qquad (13)$$

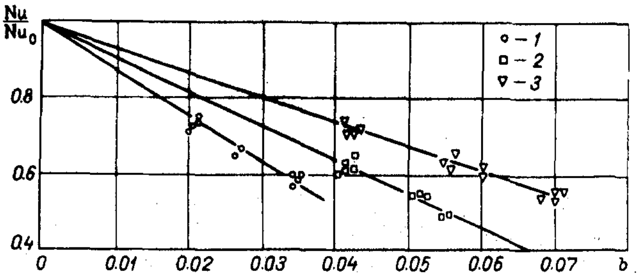
If the complex $(K + (T_w - T_1)/(T_\infty - T_w))$ varies within a small range, the relationship between $\mathrm{Nu}/\mathrm{Nu}_0$ and b is close to some linear function.

In the region of small and moderate lateral flows of material (which is characteristic precisely of the evaporation processes) the linear function $\mathrm{Nu}/\mathrm{Nu}_0 = 1 - C_1 b$, where C_1 is the function of the thermodynamic properties of the incoming material, has been observed repeatedly in experiments. Considering (13), we obtain

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_0}=\left(1+\frac{c_{p\infty}}{c_1}\,\frac{C_1}{K+\dfrac{T_w - T_1}{T_\infty - T_w}}\right)^{-1}. \qquad (14)$$

Reference [3] reports results on the experimental investigation of the processes of heat- and mass-transfer in evaporation in a turbulent boundary layer of highly volatile liquids—ethyl alcohol, benzene, and acetone. The figure shows the function $\mathrm{Nu}/\mathrm{Nu}_0 = f(b)$, from which the validity of the linear approximation adopted in the derivation of (14) is evident.

With identical values for the parameter b, liquids with the highest value for the heat of phase transition are the best coolants.



Plot of heat transfer coefficient on evaporation of various liquids versus mass-transfer parameter b: 1) alcohol; 2) acetone; 3) benzene.

The experimental results were processed in accordance with the heat-balance equation, as well as in accordance with expressions (12) and (14). The quantity C_1 was determined from experimental data and Nu_0 was taken from the Fedorov data [4]. The results of the calculation are summarized in the table.

Analysis of the table permits us to draw the following conclusions.

1) The quantities calculated from (12) and (14) differ permissibly from the experimental values. Certain deviations can be explained by fractions of nonconvective leakages of heat in the overall balance (the radiative component, the flow of heat for the heating of the model, etc., although measures were implemented to reduce these to a minimum). However, in deriving relationships (12) and (14), we assumed that all of the incoming heat was expended on the phase transition and on heating the liquid.

Another reason for the slight discrepancy between the calculated and balanced quantities might be the nonsimilarity of the dynamic and temperature fields of the boundary layer observed in our experiments, while the proportionality $Cf/2 = St$ follows from the hypothesis of similarity for the velocity and temperature fields.

2) It is obvious that the simplexes C_l/C_{pw} and $C_{p\infty}/C_l$ do not significantly change in transition from one liquid to another.

3) Our nonadiabaticity parameter $(T_w - T_l/(T_\infty - T_w)$ is very small, particularly in comparison with the criterion of phase conversion, entering the theoretical relationship (12), (13), and (14), in complex with the latter. This is quite natural if we take into consideration that the fraction of heat expended on the heating of the liquid is substantually smaller (approximately by 2 orders) than the heat expended on the phase conversion.

It follows from (12), (13), and (14) that the generalized description of the heat-transfer process in the case of evaporation in a turbulent boundary layer is possible for various liquids in which the thermodynamic conditions and properties of the transported material are accounted for by a single complex.

Our experimental data on the local and average transfer of heat under nonadiabatic conditions are described by the following expressions [3]:

$$\mathrm{Nu}_{qx} = 0.01\,\mathrm{Re}_x^{0.8} K^{0.3},$$

$$\overline{\mathrm{Nu}}_q = 0.012\,\mathrm{Re}_\infty^{0.8} K^{0.3} \tag{15}$$

and

$$\overline{\mathrm{Nu}}_{qx} = 0.0297\,\mathrm{Re}^{0.8}(1 - b_{\mathrm{H}})^9,$$

$$\overline{\mathrm{Nu}}_q = 0.0331\,\mathrm{Re}^{0.8}(1 - b_{\mathrm{H}})^9. \tag{16}$$

The resulting functions are also extended to the Fedorov [4] data which deal with the vaporization of water under conditions close to the adiabatic. The possibility of processing the results of both adiabatic and nonadiabatic vaporization with a single empirical expression confirms our assumption as to the weak effect on the processes of transport by deviation from adiabaticity in the turbulent boundary layer on evaporation from a porous wall.

NOTATION

c is the heat capacity; r is the phase-transition heat; ρ is the density; λ is the thermal conductivity coefficient; u is the velocity; $\dot{q}_w''$ is the specific heat flux; $\dot{m}''$ is the specific mass flux; T is the temperature; St is the Stanton number; Nu is the Nusselt number; K is the phase conversion number; Re is the Reynolds number; Pr is the Prandtl number. Subscripts: w denotes the wall; ∞ denotes the freestream; 0 denotes no mass transfer; l denotes liquid.

Comparison of plate heat-transfer data, for a balance and for calculations according to Eqs. (12) and (14)

Number of points	b_{H}	K	$\frac{T_w - T_l}{T_\infty - T_w}$	$\frac{c_l}{c_{pw}}$	$c_{p\infty}/c_l$	C_1	Nu/Nu_0 according to thermal balance	Nu/Nu_0 according to Eq. (12)	Nu/Nu_0 according to Eq. (14)
1	0.02845	15.493	0.1628	1.5199	0.4646	12.072	0.7285	0.6770	0.7362
2	0.03933	10.755	0.1998	1.5199	0.4646	12.072	0.6735	0.6549	0.6614
3	0.04360	7.908	0.0894	1.5199	0.4646	12.072	0.6220	0.5300	0.5878
4	0.04163	9.613	0.4273	1.6216	0.5884	6.914	0.7113	0.6779	0.7116
5	0.06054	6.184	0.3467	1.6216	0.5864	6.914	0.6082	0.6411	0.6524
6	0.07112	4.685	0.3010	1.6216	0.5884	6.914	0.5395	0.5750	0.5507
7	0.05265	6.734	−0.1565	1.4809	0.4686	8.972	0.6254	0.6128	0.6101
8	0.07458	4.969	−0.0228	1.4809	0.4686	8.972	0.5636	0.5463	0.5405
9	0.07954	4.143	+0.0168	1.4627	0.4686	8.972	0.4811	0.4840	0.4973

REFERENCES

1. A. V. Luikov, IFZh, no. 11, 1962.

2. V. P. Motulevich, collection: Physical Gasdynamics, Heat Transfer, and Thermodynamics of High-Temperature Gases [in Russian], Izd. AN SSSR, 1962.

3. R. Sh. Vainberg, IFZh [Journal of Engineering Physics], no. 7, 1967.

4. B. I. Fedorov, Author's Abstract of Candidate's Dissertation [in Russian], Institute of Heat and Mass Transfer AS BSSR, Minsk, 1964.

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